CONSUMPTION OF ELECTRODE MATERIAL IN CATHODE REGION OF AN ARC

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The formula for the evaporation rate in a vacuum [5] is often used to evaluate the temperature of the cathode spot and to construct a theory of the cathode region [1-4, 9]. It is assumed in this case that the rate of erosion (removal of material), determined from the loss of weight of the electrode, is the same as the evaporation rate. In the case where the vapor pressure close to the cathode spot is high, the rate of removal of material and the evaporation rate are not equal to one another. In this paper the model of the cathode proposed in [6] is used to obtain an expression for the rate of removal of electrode material in terms of other parameters of the cathode region. This relationship can be regarded as the boundary condition for the solution of the problem of dispersion of vapor from a cathode spot into a vacuum. Generally speaking, the determination of the removal rate requires the joint solution of the vapor dispersion problem and heat conduction problem within the electrode, which are connected by the above-mentioned boundary condition. For the construction of a theory of the cathode region without solving the vapor dispersion problem the removal rate can be determined from the relationships obtained in this paper by using experimental erosion values. The experimental data of [4, 7] for arcs on copper cathodes are treated in this way as an example, and all the cathode region parameters – current density, temperature, etc. – are determined.

It is known (see [1], for instance) that a vacuum arc is maintained by the continuous evaporation of the electrode material in the cathode region. The energy for evaporation of the electrode material is provided by the positive ions striking the cathode. The ions, in turn, are formed in the cathode region by impact ionization of atoms of the electrode material vapor by electrons emitted by the electrode and accelerated in the cathode fall region. The ionization and diffusion of ions to the electrode surface depend on the vapor pressure near the surface and the rate of removal of vapor into the interelectrode space. In view of this there will be, in the steady state, some relationship between the vapor pressure near the cathode, the rate of removal of vapor (consumption of material due to removal of vapor), current density, and current composition (ratio of electronic and ionic current components). This relationship in the general case gives the rate of removal of electrode material (erosion) in the case where all the material is removed in the form of vapor.

The rate of removal of electrode material from the cathode region is sometimes evaluated (see [2-4], for instance) by using the expression for the rate of evaporation of a material at a given temperature [5]

$$W = CT^{-1/2} \exp\left(-\frac{B}{T}\right) \tag{1}$$

Here W is the evaporation rate ($g/cm^2 \cdot sec$), T is the temperature, and C and B are constants of the material. We note that if the electrode surface temperature is known, formula (1) can be used to determine the removal rate only in the case where the vapor pressure near the surface is very low (the macroscopic vapor removal velocity v is much greater than the thermal v_T).

Since ions move out of the ionization region by diffusion [6], it is clear that the condition $v \gg v_T$ would mean that the ion component of the current on the electrode is zero and, hence, there is no supply of energy and no evaporation. In view of this, Eq. (1) cannot be used to determine the removal of electrode material in vapor form.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 3-10, September-October, 1970. Original article submitted June 4, 1970.

To determine the rate of removal of electrode material and its connection with other parameters in the cathode region, we extend the solution of [6] to the case where the material vapor in the cathode region has a velocity differing from zero.

We assume that the degree of ionization in the cathode ionization region (at a distance of the order of the ionization mean free path of the electron) is low. In this case we can assume that the vapor velocity v, the pressure p, and temperature T (and, hence, the neutral particle concentration n) in the ionization region are constant, and the ion concentration distribution is given by the diffusion equation

$$\frac{d\Gamma_i}{dx} = n\sigma_i N_{e0} e^{-n\sigma_i x}, \quad \Gamma_i = -D\frac{dn_i}{dx} + n_i v \qquad \left(D = \frac{v_T}{3n\sigma_r}\right) \tag{2}$$

Here Γ_i is the diffusion flux, n_i is the ion concentration, v_T is the thermal velocity, σ_i is the ionization cross section, N_{e0} is the electron flux from the electrode surface, and D is the ion diffusion coefficient, which depends on their resonance charge-exchange cross section σ_r. The right side of Eq. (2) is the ionization function.

We assume that the removal velocity in the ionization region is much less than the thermal velocity

$$v \ll v_T \tag{3}$$

The removal velocity, which determines the specific consumption of electrode material

$$G^* = mnv \tag{4}$$

(where m is the mass of an electrode atom), will be found when the problem is solved. In view of this the assumption (3), which was justified by the physical formulation of the problem, will have to be verified after solution.

In the case of a Maxwellian particle distribution and fulfillment of (3) the random flux of particles from the gas to the wall in the absence of a concentration gradient is given by the expression

$$\Gamma = \frac{nv_T}{4}\left[1 - 2\frac{v}{v_T} + o\left(\frac{v^2}{v_T^2}\right)\right] \tag{5}$$

In this connection it is easy to show that the boundary condition on the electrode surface (x = 0), accurate to terms of the order of $(v/v_T)^2$, is the same as in the case where v = 0. Thus, the boundary conditions for Eq. (2) have the form [6]

$$D\frac{dn_i}{dx} = \frac{1}{2}n_i v_T \quad \text{at} \quad x = 0, \qquad \frac{dn_i}{dx} = 0 \quad \text{at} \quad x = \infty \tag{6}$$

On the other hand, the balance relationship for neutral particles on the electronic surface when condition (3) holds gives

$$^1/_4 nv_T(1 - 2v/v_T) + nv = W = {}^1/_4 n_s v_T \tag{7}$$

Here n_s is the equilibrium concentration at the given temperature, and W is the evaporation rate (1). It follows from (7) that

$$\frac{n_s - n}{n} = 2\frac{v}{v_T} \tag{8}$$

Thus, the neutral particle concentration and pressure in the ionization region, accurate to terms of the order of v/v_T, are equal to their equilibrium values

$$n = n_s = \frac{p_s}{kT}, \qquad p = p_s = A\exp\left(-\frac{B}{T}\right) \tag{9}$$

Here A and B are constants of the electrode material [5].

The solution of Eq. (2) with boundary conditions (6) has the form

$$f = \frac{n_i}{n} = c[(1 + \beta) - e^{-y}], \qquad \frac{df}{dy} = ce^{-y}, \qquad y = n\sigma_i x$$
$$c = \frac{2}{\alpha\beta + 2a}, \quad a = \frac{nv}{N_{e0}}, \quad \alpha = \frac{nv_T}{N_{e0}}, \quad \beta = \frac{2}{3}\frac{\sigma_i}{\sigma_r} \tag{10}$$

When $a = 0$ this solution becomes the solution in [6].

Using solution (10) and determining the ion flux (2), we can easily obtain the fraction of electron current on the electrode surface

$$S = \frac{I_e}{I} = \frac{N_{e0}}{N_{e0} + |\Gamma_{i0}|} = \frac{1}{1 + |\Gamma_{i0}|/N_{e0}} \approx \frac{1}{2 - |\Gamma_{i\infty}|/N_{e0}} \equiv \frac{1}{1+b} \tag{11}$$

$$\Gamma_{i0} = -D\frac{dn_i}{dx}\Big|_0 + n_{i0}v = -\frac{nv_T}{2}c\beta\left(1 - 2\frac{v}{v_T}\right)$$

$$\Gamma_{i\infty} = n_{i\infty}v = c(1+\beta)nv$$

$$b \equiv |\Gamma_{i0}|/N_{e0} = \frac{\alpha\beta}{2}c\left(1 - 2\frac{a}{\alpha}\right) \tag{12}$$

If there is no removal of mass (v = 0) $|\Gamma_{i0}| = N_{i0}$ and S = 0.5 [6]. To connect the removal G = nv = G*/m with the other parameters we use the energy equation. If the electrode surface temperature (in the region of the arc spot) is such that the vapor saturation density is much less than the density of the liquid metal (for metal cathodes), the velocity of the evaporation front can be regarded as zero, and the consumption of vaporized material can be determined from the energy equation at the evaporation front

$$e(U_c + U_i - \Phi)|\Gamma_{i0}| = \lambda G + \lambda_1\left[\frac{dT}{dx}\right]_0 + q \equiv \lambda G + q_T + q \tag{13}$$

Here e is the electron charge; U_c is the cathode fall; λ, λ_1 are the heat of evaporation and thermal conductivity of the electrode material; q is the total energy loss due to radiation, destruction outside the vapor phase, etc.; U_i is the vapor ionization potential; and Φ is the work function of the electrode material. The quantity q_T is the heat flow into the electrode due to heat conduction.

From (13) we obtain

$$b = \frac{|\Gamma_{i0}|}{N_{e0}} = \gamma a + Q$$

$$\gamma \equiv \frac{\lambda}{e(U_c + U_i - \Phi)}, \quad Q \equiv \frac{q_T + q}{e(U_c + U_i - \Phi)N_{e0}} \tag{14}$$

We note that in (13) and (14) the energy accommodation coefficient and the ion neutralization are taken as unity. The generalization of these relationships to include the case of coefficients differing from unity presents no difficulties (see [1], for instance), but the use of the obtained expressions for accurate calculations requires a knowledge of these coefficients.

From (12) and (14) we obtain an equation for a

$$\gamma a + Q = \frac{\alpha\beta}{2}c\left(1 - 2\frac{a}{\alpha}\right) \tag{15}$$

The system of equations (8)-(11), (15), in conjunction with the emission equations (see [1, 2], for instance) and the relationship

$$eN_{e0} = jS \tag{16}$$

enables us to determine all the parameters in the ionization region, particularly the consumption of electrode material in the vapor phase

$$G^* = maN_{e0} = \frac{m}{e}ajS \tag{17}$$

if the energy loss Q and spot surface temperature are known. In the special case where energy is removed solely by heat conduction and evaporation, the relationship between T and Q is given by the solution of the equation of heat conduction within the electrode and the problem then includes only one free parameter (e.g., the temperature or removal velocity).

To close the system we can use, for instance, experimental data for the rate of removal of electrode material (see [1, 3, 4, 7] for instance) in relation to the total arc current I.† In this case, assuming the spot homogeneous, we obtain from (17) the following relationship for a:

$$a = \frac{eG^{**}}{mjS\Sigma} = \frac{eG^{**}}{ImS} \qquad \left(\Sigma = \frac{I}{j}\right) \tag{18}$$

Here G** = G**(I) is the experimentally determined relationship between the total removal (mass) of material and the total arc current, and Σ is the area of the cathode spot.

The results of calculation of the parameters in the cathode region from the above relationships can be used to calculate other "macroscopic" characteristics of an arc and for a subsequent comparison of the results of calculation with experimental data.

For instance, it is easy to calculate the force acting on the arc cathode. In fact, the specific momentum of a jet of electrode material vapor is, in view of (3),

$$J \equiv p + G^*v \approx p_s \tag{19}$$

Thus, the specific momentum of the jet depends only on the pressure in the ionization region, which is practically the same as the saturation vapor pressure at the given surface temperature.

Using (19) we obtain a relationship connecting the reaction force f on the cathode with the arc current:

$$f = (p_s + J_i)\Sigma = (p_s + J_i)\frac{I}{j} \tag{20}$$

Here J_i is the momentum transmitted to the electrode surface by the ions "bombarding" it. If the momentum accomodation coefficient is taken as unity, then

$$J_i = |\Gamma_{i0}| v_i = |\Gamma_{i0}| \left(\frac{eU_c}{m_i}\right)^{1/2} \tag{21}$$

Using the solution of (10), (11), and condition (3), we can easily estimate the relative magnitude of the terms in (20):

$$\frac{J_i}{p_s} \sim \frac{|\Gamma_{i0}| v_i}{p_s} \sim \frac{n_{i0}}{n}\frac{v_i}{v_T} \tag{22}$$

For temperatures of the order of several thousand degrees and $U_c \sim 10$ V the ratio $v_i/v_T \sim 1$.

On the other hand,

$$\frac{n_{i0}}{n} \sim \frac{\beta\alpha^*}{1-\alpha^*} \ll 1 \qquad \left(\alpha^* \equiv \frac{n_{i\infty}}{n_{i\infty}+n}\right)$$

Here α^* is the degree of ionization at infinity.

Thus, the second term in Eq. (20) can be neglected and

$$f = p_s\Sigma = p_s\frac{I}{j} \tag{23}$$

To evaluate the various parameters in the cathode region and to compare the results of calculation with experimental data, we treated the results of the experimental investigations [4, 7] by using the theory developed above. An arc on a copper cathode was investigated in these works. In [4] the erosion of the electrode in the current range I = 200–700 A was measured, and in [7] the erosion and reaction force on the cathode of currents I = 11–32 A were measured.

†In the general case the missing relationship for the removal velocity will be obtained from a joint solution of the heat problem within the electrode and the gasdynamic problem of dispersion of vapor into a vacuum. Since such solutions are not available the calculation of the parameters in the spot necessitates the use of experimentally obtained relationships – data for the erosion, or the force acting on the cathode, and so on.

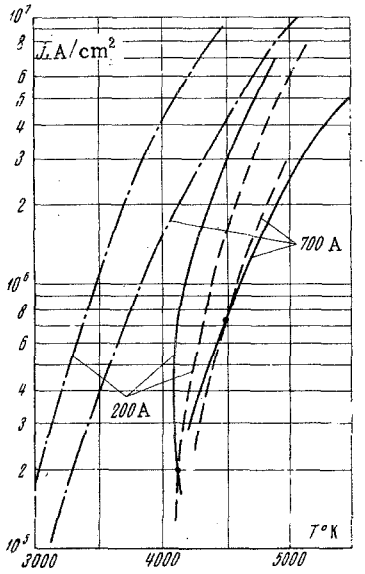

Fig. 1

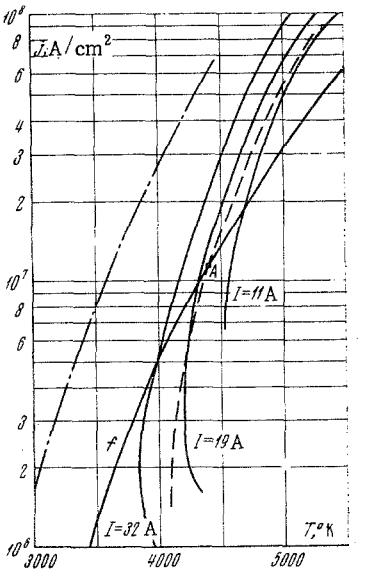

Fig. 2

In the calculations the spot was assumed to be steady and homogeneous and the temperature in the spot was assumed to be equal to the temperature at the center of a spot of area $\Sigma = I/j$, situated on the surface of a semiinfinite solid. In addition to relationships (8)-(15) we used the solution of the heat conduction equation in a solid [8]

$$T = \frac{q_T}{k}\left(\frac{\Sigma}{\pi}\right)^{1/2} = \frac{q_T}{k\sqrt{\pi}}\left(\frac{I}{j}\right)^{1/2} \tag{24}$$

Here k is the thermal conductivity of copper, and q_T is determined from (13) with q = 0.

Figure 1 shows the results of calculations based on the data of [4]. The continuous curves were obtained by using the relationships (8)-(13), (15), and (18). We used the experimental data relating the consumption of cathode material with the arc current (18), but we did not use the relationship connecting the emission current with the other parameters. In this way we obtained curves in the jT plane which we will call "consumption curves." The working point on these curves corresponds to their intersection with the "emission" curve $j = j(S, T)$, which represents the emission law – the dashed curves in Fig. 1. These curves were plotted by converting the data of [2] with the aid of the curves $S = S(j, T)$ obtained from formulas in that paper. Since the obtained curves are similar to one another and intersect at small angles the obtained value of current density j and spot temperature T is infinite. In view of the assumptions embodied in the theory we can say with confidence that practically all the points on the "consumption" curves – the continuous lines – are consistent with thermionic and field emission. Additional data are required to isolate the working region of current densities.

Figure 2 shows the consumption curves based on the data of [7] for different arc currents (I = 32, 19, 11 A). This figure also shows the emission curve and the force (f) curve obtained by treatment of the data for the force on the cathode with the aid of Eq. (23).

Because of the linear dependence of the force and consumption on the arc current the emission and force curves are independent of the arc current. Hence, the working point should correspond to their intersection – point A. The consumption curves for different currents should also pass through this point. Figure 2 shows that although the consumption curves are close to the point of intersection of the emission and force curves they do not intersect one another at this point. An examination of the experimental points in [7] shows that they are scattered about the approximating straight line established in the calculation. This scatter may be responsible for the shift of the points of intersection of the force and consumption curves away from point A.

To illustrate by how much the erosion rate can differ from the evaporation rate (1) we show in Figs. 1 and 2 the curves given by the following relationship (dot–dash curves):

$$\frac{G^*j}{IW} = \frac{G^*j}{I}\frac{\sqrt{T}}{C}\exp\frac{B}{T} \tag{25}$$

A comparison of these curves with the consumption curves shows that the specific (per unit area) erosion rate W, determined from formula (1) for a given spot temperature, can differ considerably from its real value G*j/I [this ratio with T = const is equal to the ratio of the ordinates of curves (25) and the consumption curves at the given temperature]. On the other hand, a determination of the temperature in the spot from Eq. (1) for a given current density [4, 9] gives a value 500-1000° below its actual value (this difference is given by the difference in the abscissas of the consumption curves and (25) with j = const).

An examination of the curves in Figs. 1 and 2 shows that the current density in the spot at low currents is much higher than at higher currents. As distinct from the current density, the temperature in the spot does not vary a great deal and lies in the range 4000-4500°K.

Using Eqs. (9)-(17) obtained above it is easy to calculate the macroscopic gasdynamic parameters close to the electrode surface in the cathode spot – the velocity v, pressure p, density n, degree of ionization $\alpha^* = n_{i\infty}/(n_{i\infty} + n)$ – as functions of the temperature and current density. These parameters relate to a diffusion region of the order of several electron-atom or atom-atom mean free paths (D layer). For the solution of the gasdynamic (magnetohydrodynamic, to be more precise) problem of dispersion of vapor outside the D layer the values of the parameters at its outer boundary – the solution of (9)-(17) – must be used as boundary conditions. From the viewpoint of the general problem, comprising the problem of heat conduction in a solid and the problem of dispersion of vapor into a vacuum (or low-pressure medium), the D layer is a discontinuity surface, the structure of which (distribution of parameters in the D layer) is divided into a space-charge layer and a quasineutral region, represented by Eq. (2) or similar diffusion equations, where the change in concentration of neutrals is important.

In specific calculations involving the experimental data described above the spot was assumed homogeneous in the sense that all the parameters were assumed to be independent of the coordinates in the plane of the spot and had the same values as at the center of the spot. Hence, the obtained values of the gasdynamic parameters must be regarded as average values. To illustrate this Table 1 gives the values (calculated) of the parameters characterizing the cathode region of the discharge and corresponding to the working points on the curves in Figs. 1 and 2.

The data in Table 1 show that condition (3) is satisfactorily fulfilled, but the assumption of a low degree of ionization is not valid at the outer boundary of the D layer. This necessitates, generally speaking, an improvement of the diffusion for the D layer.

We note in conclusion that the vapor velocities close to the electrode surface, obtained in the calculation, are significantly lower than the thermal velocity (see Table 1) and have low absolute value (~10^3 cm/sec). This fact can be shown to contradict the experimental data, which give ~10^6 cm/sec from direct measurements, and these values correspond in magnitude of jet momentum with force measurements (see [1], for instance). This may be due to the fact that the velocities were measured in the jet at a considerable distance from the electrode surface. The vapor jet may be accelerated by Joule heat and a change in its cross-sectional area. The question of the possibility of obtaining high velocities in the jet when the velocities at the electrode surface are low may be solved by an analysis of the magnetogasdynamic problem of vapor dispersion. On the other hand, the momentum of the jet during dispersion may vary slightly and then its measured value will lead to a fairly accurate value of the reaction force on the cathode. Another explanation of the high values of the measured velocity may be the pressure of fast ions in the jet [10].

TABLE 1

I	$\frac{j}{10^5}$	$\Sigma\cdot10^3$	S	$\frac{j_i}{10^5}$	$\frac{j_e}{10^5}$	$\frac{p}{10^3}$	$\frac{n}{10^{20}}$	T	$\frac{v}{10^3}$	$\frac{v_T}{10^5}$	α^*
700	6	1.5	0.83	1	5	15	3.3	4500	4.3	1.2	0.42
200	2	1	0.6	0.8	1.2	8	1.8	4100	1	1.1	0.63
19	10	2.4	0.85	1.2	6.8	10	2.2	4250	7	1.17	0.80

The author thanks V. I. Rakhovskii and I. I. Beilis for numerous discussions, without which the present paper would probably not have been completed.

LITERATURE CITED

1. I. G. Kesaev, Cathode Processes in an Electric Arc [in Russian], Nauka, Moscow (1968).
2. T. H. Lee and A. Greenwood, "Theory for the cathode mechanism in metal vapor arcs," J. Appl. Phys., 32, No. 5 (1961).
3. V. I. Rakhovskii, "An investigation of erosion of refractory electrodes in a high-current vacuum arc," Zh. Tekh. Fiz., 34, No. 11 (1964).
4. V. V. Kantsel', T. S. Kurakina, V. S. Potokin, V. I. Rakhovskii, and L. G. Tkachev, "Effect of thermophysical parameters of material on electrode erosion in a high-current vacuum discharge," Zh. Tekh. Fiz., 38, No. 6 (1968).
5. S. Dushman, Scientific Foundations of Vacuum Technique, Wiley, New York (1962).
6. I. I. Beilis, G. A. Lyubimov, and V. I. Rakhovskii, "The electric field on the electrode surface in an arc cathode spot," Dokl. Akad. Nauk SSSR, 188, No. 3 (1969).
7. R. Tanberg, "On the cathode of an arc drawn in vacuum," Phys. Rev., 35, No. 9 (1930).
8. H. Carslaw and J. Jaeger, Heat Conduction in Solids, Clarendon Press, Oxford (1959).
9. I. I. Beilis and V. I. Rakhovskii, "Theory of cathode mechanism of arc discharge," Teplofiz. Vys. Temp., 7, No. 4 (1969).
10. W. D. Davies and H. C. Miller, "Analysis of electrode products emitted by dc arcs in vacuum ambient," J. Appl. Phys., 40, No. 5 (1969).